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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

20 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

2226 – 5227

9 pages

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Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 25]

This question is about using statistical techniques to analyse incorrect calls (errors) made in high-pressure situations by basketball referees.

Gabriel is investigating whether a basketball referee makes more or fewer incorrect calls as the pressure increases towards the end of a close game.

Gabriel collects data from the National Basketball Association (NBA).

He finds data on the number of incorrect calls made in the final two minutes of games, in which the teams are within three points of each other. Gabriel uses these data in each of his tests.*

He selects four teams from the 30 teams in the NBA, listed in alphabetical order.

(a) Explain how Gabriel might select his sample of four teams using systematic random sampling. [2]

* *NBA officiating last two minute reports archive*

(This question continues on the following page)

(Question 1 continued)

In the sample taken by Gabriel there is a total of 156 incorrect calls. Gabriel divides the final two minutes of the game into 20-second intervals and records the interval in which each of the 156 incorrect calls happen.

Gabriel decides to use a χ^2 goodness of fit test, at the 5% significance level, with the following hypotheses:

H_0: Incorrect calls are equally distributed across the given intervals,

H_1: Incorrect calls are not equally distributed across the given intervals.

The data is summarized in the following table, where t is the number of seconds from the end of the game.

***t* (seconds)**	$0 \le t < 20$	$20 \le t < 40$	$40 \le t < 60$	$60 \le t < 80$	$80 \le t < 100$	$100 \le t < 120$
Number of incorrect calls	41	31	23	22	20	19

(b) (i) Find the expected number of incorrect calls per 20-second interval under the null hypothesis. [2]

(ii) State the number of degrees of freedom for the test. [1]

(iii) Find the value of χ^2_{calc}. [2]

(iv) Hence state the conclusion of the test, given that the critical value is 11.1. Justify your answer. [2]

(v) State another feature of Gabriel's data that supports the theory that the number of incorrect calls increases as the pressure on the referee increases. [1]

(This question continues on the following page)

(Question 1 continued)

Gabriel then wonders if the incorrect calls might benefit the team playing at home (the home team) more than the team playing away from home (the away team). He finds the total number of incorrect calls given in the final two minutes of close games for all 30 NBA teams in a season, and decides he will test the data using the following hypotheses:

H_0: The probability of an incorrect call benefitting the home team is equal to 0.5,

H_1 : The probability of an incorrect call benefitting the home team is greater than 0.5.

His data for incorrect calls is shown in the following table.

Calls benefitting the home team	Calls benefitting the away team
695	637

Gabriel decides to use the binomial distribution to test these hypotheses, at the 5% significance level.

(c) (i) State one assumption that Gabriel is making. [1]

(ii) Find the p-value for the test. [3]

(iii) State the conclusion of the test, justifying your answer. [2]

Martina suggests to Gabriel that, as some teams have more passionate fans than others, the number of incorrect calls that favour the home team might depend on their stadium.

Gabriel decides to do a χ^2 test for independence for the four teams he has selected: Detroit, Brooklyn, Cleveland and Philadelphia. The distribution of incorrect calls favouring the home and away teams are shown in the following table.

		Team playing at home			
		Detroit	**Brooklyn**	**Cleveland**	**Philadelphia**
Favourable incorrect calls	**Home team**	24	23	18	23
	Away team	9	16	21	22

(d) (i) State, in context, the hypotheses for this test. [2]

(ii) By finding the p-value, verify that the test is not significant at the 5% level. [2]

(This question continues on the following page)

(Question 1 continued)

Finally, Gabriel decides to test whether his local team, Brooklyn, receive more favourable incorrect calls when playing at home than when playing away.

He reviews the previous five seasons and finds seven opponents for which he has home and away data. He collates these in the following table.

Opponents	**Utah**	**Philadelphia**	**Atlanta**	**Charlotte**	**Indiana**	**Dallas**	**Toronto**
Advantage gained at home	2.2	0.1	0.1	−1.1	−2.3	1.8	−0.8
Advantage gained away	0.3	−0.9	0.1	−1.0	−1.2	−1.1	−0.7

The numbers in the table are the average difference between the favourable incorrect calls for Brooklyn, minus the unfavourable incorrect calls.

For example, the value of 2.2 in the first column means that, when playing Utah at home, Brooklyn had on average 2.2 more favourable incorrect calls than unfavourable incorrect calls.

This is defined as an "advantage gained" of 2.2.

Let μ_D be the population mean of Brooklyn's advantage gained at home minus its advantage gained away.

(e) Use a paired t-test, at the 5% significance level, to determine whether these data provide evidence that the mean advantage gained at home is greater than the mean advantage gained away.

State the hypotheses in terms of μ_D, find the p-value and state the conclusion of the test. You may assume all the conditions necessary for the test have been met. [4]

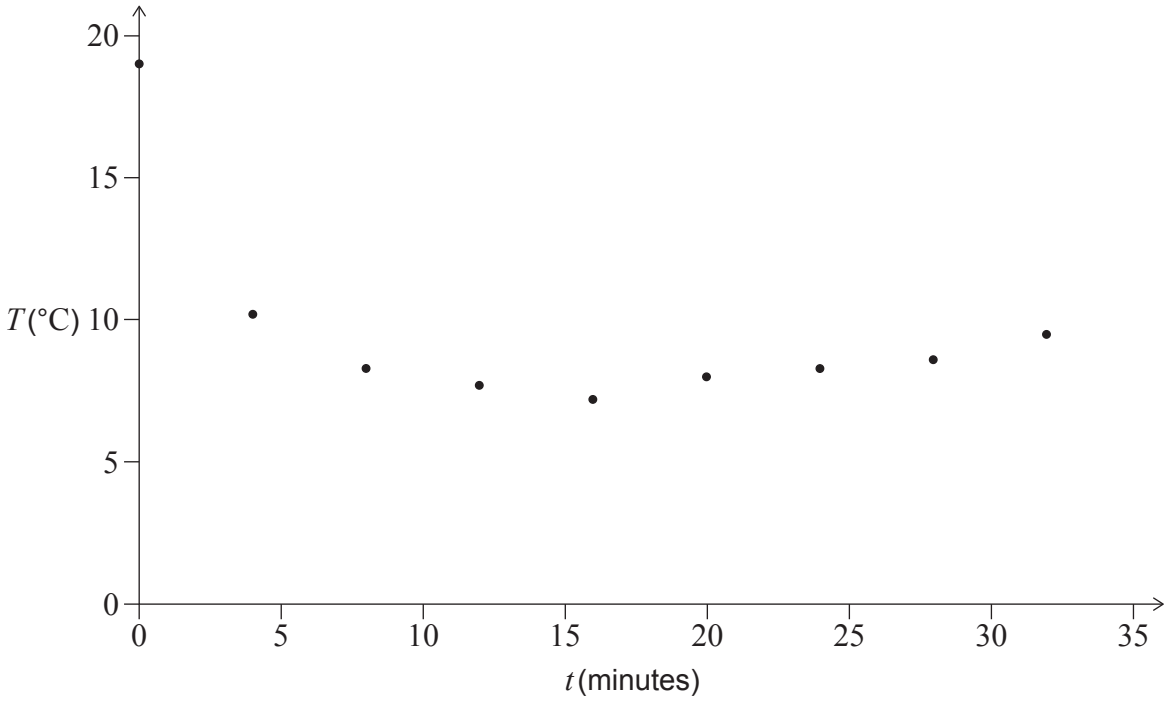
(f) Justify a possible conclusion Gabriel might reasonably reach from the results of his tests as to whether there is a home advantage in the incorrect calls made by referees in the last two minutes of close games. [1]

2. [Maximum mark: 30]

This question considers different models for the temperature change of water when an ice cube is added and left to melt.

Aisha placed an ice cube into a glass of water at room temperature and recorded the temperature of the water at four-minute intervals. The temperature of the water, T, in degrees Celsius, after t minutes is shown in the following table and diagram.

t **(minutes)**	0	4	8	12	16	20	24	28	32
T **(°C)**	19	10.2	8.3	7.7	7.2	8.0	8.3	8.6	9.5



Aisha decides she will model the data with a quadratic model.

(a) (i) Using the diagram, state one reason why Aisha might choose a quadratic model. [1]

(ii) Find the equation of the least squares quadratic regression curve for the data. [3]

(b) Explain, from the context of the question, why the quadratic model is unlikely to be a good match for larger values of t. [2]

(This question continues on the following page)

(Question 2 continued)

From research, Aisha decides a better model will be in the form

$$T = c - at^{\frac{1}{2}}e^{-bt}, \text{ where } a, b, c \in \mathbb{R}^+.$$

She decides to use her value of T when $t = 0$ to find c.

(c) Show that $c = 19$. [1]

Aisha decides to find a value for each of the parameters a and b, using two different methods.

The first method is to assume that the minimum for the function T occurs at the point $(16, 7.2)$.

(d) (i) Find an expression for $\frac{dT}{dt}$ in terms of a, b and t. [3]

(ii) Hence show that $b = \frac{1}{32}$. [4]

(iii) By substituting $(16, 7.2)$ into the equation for T, find the value of a. [2]

(iv) Hence write down the equation for T obtained by Aisha when using this method. [1]

(This question continues on the following page)

(Question 2 continued)

The second method Aisha uses is to transform the data and use linear regression.

To do this, she first rearranges the equation for T into the form

$$\frac{19-T}{t^{\frac{1}{2}}} = a\mathrm{e}^{-bt}.$$

Aisha then lets $\frac{19-T}{t^{\frac{1}{2}}} = x$, to form the equation $x = a\mathrm{e}^{-bt}$.

She calculates values of x from her data, as shown in the following table.

t	4	8	12	16	20	24	28	32
T (°C)	10.2	8.3	7.7	7.2	8.0	8.3	8.6	9.5
x	4.40	3.78	3.26	q	2.46	2.18	1.97	1.68

(e) Find the value of q. [1]

(f) (i) Find the equation of the least squares regression line of $\ln x$ on t. [3]

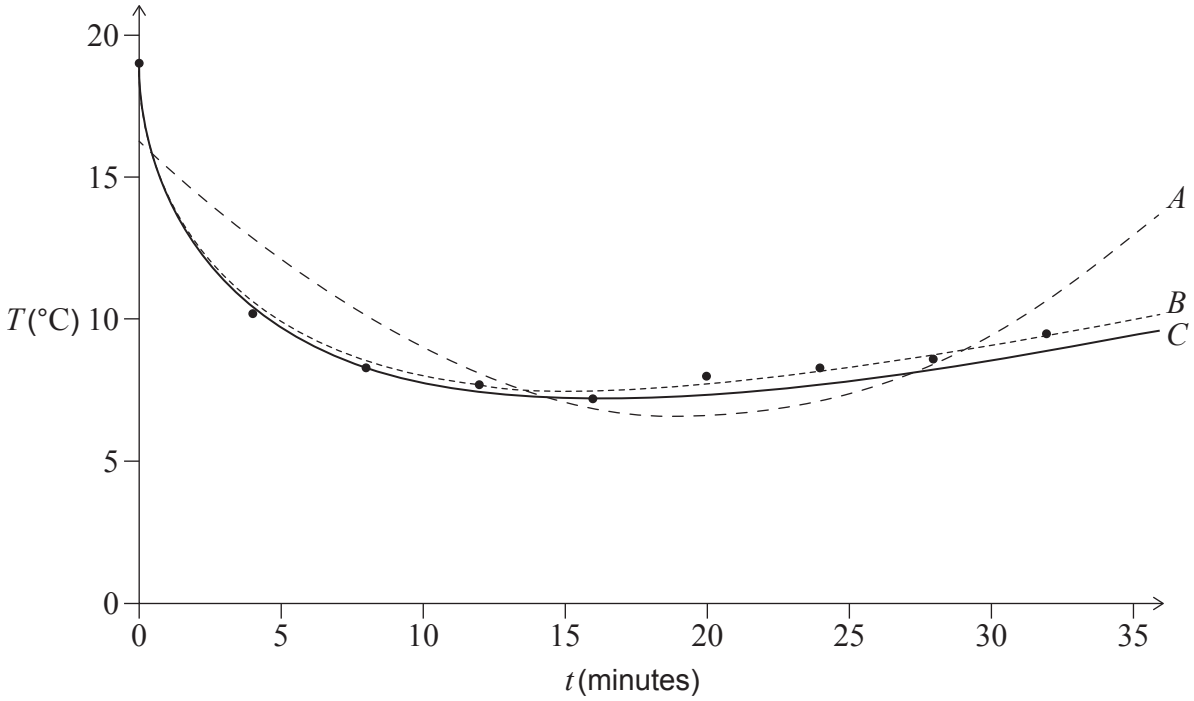
(ii) Hence find the value of a and the value of b, obtained by Aisha when using this second method. [4]

(iii) Write down the equation for T obtained by Aisha when using this second method. [1]

(This question continues on the following page)

(Question 2 continued)

The curves in the following diagram represent Aisha's three models found in parts (a)(ii), (d)(iv) and (f)(iii).



(g) State, with a reason, which graph, A, B or C, represents the model found in part

(i) (a)(ii). [2]

(ii) (f)(iii). [2]

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References:

1. NBA, 2024. NBA officiating last two minute reports archive. *NBA Official*, [online] Available at: https://official.nba.com/nba-officiating-last-two-minute-reports-archive/ [Accessed 16 January 2025].